Relation between symmetry breaking and the anomalous Josephson effect

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We report a numerical study and symmetry analysis of the anomalous Josephson current in junctions coupled with a two-dimensional electron gas (2DEG). Taking the 2DEG with the coexistence of spin-orbit coupling (SOI) and Zeeman field as an example, we determine the symmetry criterion for the appearing of an anomalous supercurrent at zero phase difference. When the Zeeman field is unsuitably oriented with SOI, the system possesses some symmetries which make the anomalous supercurrent zero and these symmetries are identified here. The symmetry criterion for the anomalous Josephson effect found here is general and suitable for any other junctions contacted with two conventional *s*-wave superconductors, for example, the superconductor- (S-) ferromagnet (F) hybrid system and the S/F/S junction on the surface of topological insulator.

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I. INTRODUCTION

The supercurrent in a Josephson junction vanishes when the phase difference between the two superconducting leads is zero.[1] In the anomalous Josephson effect, a supercurrent I_a flows even at zero phase difference. This anomalous effect has been predicted for Josephson junctions of unconventional superconductors[2–5] but so far there is no experimental verification. Recent studies have shown that the anomalous Josephson effect can also be found in junctions with conventional *s*-wave superconductors if both spin-orbit interaction (SOI) and a suitably oriented Zeeman field are present in the coupling layer of the junction.[6–10] Similar anomalous supercurrent are also found in superconductor- (S-) ferromagnet (F) hybrid structure without SOI (Refs. 11 and 12) and the S/F/S junctions on the surface of topological insulator.[13] The chiral Dirac fermions on the surface of topological insulator play the role of the SOI term and the anomalous supercurrent in the S/F hybrid structure is related to the triplet superconducting correlation[14–16] in the F layer. However, there is a lack of proper understanding of the responsible mechanism and a serious controversy over the existence conditions for the anomalous supercurrent. Zazunov *et al.*[10] studied a junction with a two-level quantum dot and found three necessary conditions for the existence of an anomalous Josephson current. Besides SOI and a suitably oriented Zeeman field, the third necessary condition is that the quantum dot is a chiral conductor. It is useful to point out here that the results in Refs. 11 and 12 show that SOI is not necessary for the existence of the anomalous current. Reynoso *et al.*[7] have found an anomalous Josephson current in a quantum point contact with SOI and a magnetic field. In these two studies quantum confinement is present. Using the phenomenological Ginzburg-Landau and the quasiclassical Eilenberger approach, Buzdin[8,9] shows that anomalous Josephson current exists when there are only ferromagnetic interaction and SOI, which is not in agreement with Ref. 10.

From the viewpoint of symmetry, the anomalous supercurrent is naturally related to the breaking of time-reversal symmetry (TRS).[8,10] The anomalous supercurrent is zero when the system has TRS.[1] However, in junctions with ferromagnetic coupling with no TRS, there is no anomalous supercurrent.[1,17–19] This means that the breaking of TRS is a necessary but not a sufficient condition for the anomalous supercurrent. In view of the existing controversies, there is a need of a systematic investigation into the origin of the anomalous supercurrent, which is found here related to symmetry breaking in the junction.

In the present work, we present a numerical investigation using a rigorous quantum-mechanical formalism and a symmetry analysis of the Andreev reflection coefficients in Josephson junctions coupled with a two-dimensional electron gas (2DEG). Our results show that quantum confinement is not a necessary condition for an anomalous supercurrent if both SOI and a suitably oriented Zeeman field are present in the 2DEG. A previous study did not obtain the anomalous supercurrent for a similar system because it has used the quasiclassical approximation.[20] More importantly, we obtain the symmetry criterion for the anomalous supercurrent, which are useful guidelines for orienting the Zeeman field as well as designing new junctions with anomalous supercurrent and superconducting rectifiers.[21,22]

II. MODEL AND NUMERICAL RESULTS

We consider a Josephson junction consisting of two conventional *s*-wave BCS superconductors coupled with a 2DEG. Both Rashba SOI (RSOI) and Dresselhaus SOI (DSOI) can be present in the 2DEG and a Zeeman field is also present. In this model, the system lies in the x-y plane and the interfaces, located at $x=0,L$, respectively, are perpendicular to the x axis, which is the current flow direction. The effective Hamiltonian of the system is given by[23,24]

$$H=\begin{pmatrix} \epsilon_k+h_z & h_{so}^*+h_{xy}^* & 0 & \Delta(x) \\ h_{so}+h_{xy} & \epsilon_k-h_z & -\Delta(x) & 0 \\ 0 & -\Delta^*(x) & -\epsilon_k-h_z & h_{so}-h_{xy} \\ \Delta^*(x) & 0 & h_{so}^*-h_{xy}^* & -\epsilon_k+h_z \end{pmatrix},$$

where $\epsilon_k=\frac{\hbar^2}{2m}(k_x^2+k_y^2-k_F^2)$ with k_F the Fermi wave number, $h_{so}=-i\alpha(x)(k_x+ik_y)-\beta(x)(k_x-ik_y)$ with $\alpha(x)=\alpha[\Theta(x)-\Theta(x$

$-L)]$, $\beta(x)=\beta[\Theta(x)-\Theta(x-L)]$ the strengths of RSOI and DSOI in the 2DEG layer, and $h_z=h(x)\cos\theta_m$, $h_{xy}=h(x)\sin\theta_m e^{i\varphi_m}$ with $h(x)=h[\Theta(x)-\Theta(x-L)]$ the strength and the direction (θ_m,φ_m) in the spherical coordinates of the Zeeman field, $\Delta(x)=\Delta[\Theta(-x)e^{i\varphi/2}+\Theta(x-L)e^{-i\varphi/2}]$ describes the pair potential with Δ the bulk superconducting gap and $\varphi=\varphi_L-\varphi_R$ the macroscopic phase difference of the two superconductor leads. The temperature dependence of the magnitude of Δ is given by $\Delta(T)=\Delta(0)\tanh(1.74\sqrt{T_c/T-1})$.[25] Since the momentum component in the y direction is conserved, the wave function can be written in the form $\Psi(x,y)=\psi(x)e^{ik_y y}$ and the Bogoliubov–de Gennes equation can be easily solved for the superconductor leads and the 2DEG layer, respectively. The scattering problem can be solved by considering the boundary conditions at the interfaces, i.e., the continuity of the wave functions and the step change of their derivatives across the interfaces.[26]

The stationary Josephson current can be expressed in terms of the Andreev reflection amplitudes by using the temperature Green's function formalism[27]

$$I(\varphi,\theta)=\frac{e\Delta k_B T}{2\hbar}\sum_{\omega_n,\sigma}\frac{1}{\Omega_n}\frac{k_n^+ + k_n^-}{\sqrt{k_n^+k_n^-}}(r_{h\bar{\sigma}e\sigma n}-r_{e\bar{\sigma}h\sigma n}), \qquad (1)$$

where k_n^+ and k_n^- are obtained from the wave vectors for electron (k_+) and hole (k_-) by the analytic continuation $E\to i\omega_n$, $r_{h\bar{\sigma}e\sigma n}$ $(r_{e\bar{\sigma}h\sigma n})$ is analytic continuation of the Andreev reflection amplitude $r_{h\bar{\sigma}e\sigma}$ $(r_{e\bar{\sigma}h\sigma})$ from an electronlike (a holelike) to a holelike (an electronlike) quasiparticle with σ representing the spin and $\bar{\sigma}=-\sigma$. It is easy to note that all the propagating modes which are related to the Andreev reflection amplitudes have been normalized with their probability current.[24] The Matsubara frequencies are $\omega_n=\pi k_B T(2n+1)$, $n=0,\pm1,\pm2,\ldots$, and $\Omega_n=\sqrt{\omega_n^2+\Delta^2}$. The incident angle θ is related to k_y through $k_y=k_F\sin\theta$. By integrating over the incident angle, the whole Josephson current is obtained as

$$J(\varphi)=\int_{-\pi/2}^{\pi/2}I(\varphi,\theta)\cos\theta d\theta. \qquad (2)$$

Figure 1 shows the anomalous current-phase relations obtained numerically for the junction. Various combinations of the strengths of SOI and the Zeeman field are considered. For the figure, the pair potential is chosen as $\Delta/\mu=10^{-3}$ with μ the chemical potential and the physical quantities are expressed in the dimensionless form. The unit of the strength of SOI is $\hbar^2k_F/2m$. The unit for the Zeeman field is μ and the length has the unit of $1/k_F$. We can clearly see that an anomalous Josephson current appears at zero phase difference for all the curves. The result of a pure DSOI is absent because it is the same as that of a pure RSOI when the Zeeman field points along the x axis instead of the y axis. When both RSOI and DSOI are present, we only present the result for the special situation where the two kinds of SOI have the same strength. In this special situation, the results are the same for the Zeeman field pointing along either the x axis or the y axis. Figure 2 shows the dependence of the absolute anomalous supercurrent on the direction of the Zeeman field in the x-y plane. The z component of the Zeeman field is ignored because all the anomalous supercurrents vanish if the Zeeman field points along the z axis. When only RSOI is present, the anomalous supercurrent is pronounced for the Zeeman field pointing along the y axis but vanish for the Zeeman field pointing along the x axis. On the other hand, the situation is the reverse for a pure DSOI. In the special case where RSOI and DSOI have the same strength, the eigen spin axis of the SOI is in the x-y plane and makes an angle of $\frac{\pi}{4}$ with the x axis. The supercurrent depends only on the misorientation angle between the direction of the Zeeman field and this eigen spin axis. The anomalous supercurrent is zero when the misorientation angle is 0 or $\frac{\pi}{2}$.

To realize the value of h considered here, one can reduce the Fermi energy in the 2DEG junction (but without the

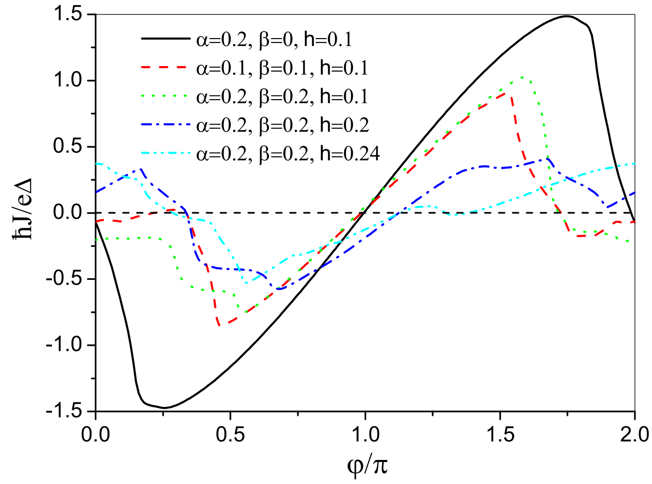

FIG. 1. (Color online) The total Josephson currents $J(\varphi)$ plotted as a function of the phase difference φ for various combinations of the strengths of SOI and the Zeeman field. The Zeeman field points along the y axis for all the curves. The length of the 2DEG layer $k_FL=10\pi$ and the temperature $T/T_c=0.01$ with T_c being the critical temperature.

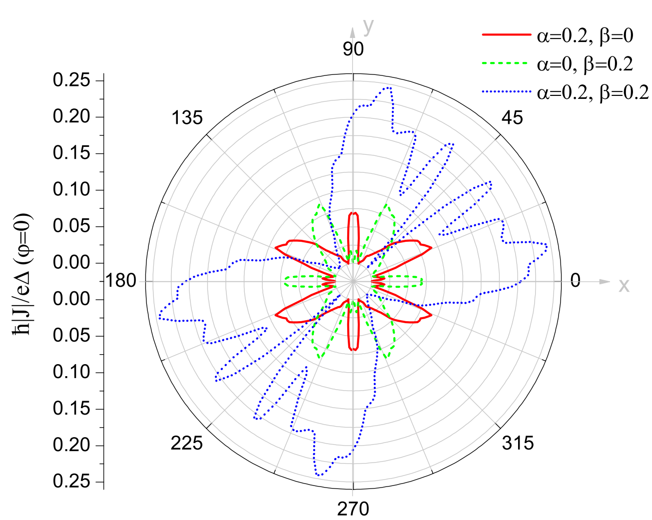

FIG. 2. (Color online) The absolute values of the anomalous Josephson currents as functions of the direction angle φ_m of the Zeeman field for various combinations of the strengths of RSOI and DSOI. The Zeeman field lies in the x-y plane, i.e., $\theta_m=\pi/2$, and with the strength $h=0.1$. The other parameters are the same as those in Fig. 1.

quantum point-contact potential) studied by Reynoso *et al.*[7] for the realization of the anomalous Josephson current. As a result, Δ/E_F ratio is increased to 0.5. We also found that the anomalous current still exists for higher Δ/E_F ratios (up to 0.5). It is also possible to find the anomalous current in a junction built from a superconductor, semiconductor, and ferromagnetic insulator, which induce the Zeeman splitting by proximity effect. These materials form the heterostructure proposed by Sau *et al.*[28] for the realization of Majorana Fermion modes. The required parameters can also be obtained by adjusting the Fermi energy. We estimated the critical currents in these structures. They are similar in magnitude to those found in other studies (e.g., Refs. 7 and 29) and therefore can be measured using the radio-frequency method as described in Refs. 29–31.

III. SYMMETRY ANALYSIS

For normal electron transport in systems without superconductors, symmetry analysis has been studied in both spin-independent[32,33] and spin-dependent transports.[34–37] In the following, we apply the method described in Ref. 36 to charge transport in Josephson junctions. For our junctions coupled with a 2DEG, the total scattering matrix S of the system can be written as follows:

$$\begin{pmatrix} b^L_{n_L\sigma_L} \\ a^R_{n_R\sigma_R} \end{pmatrix} = \begin{pmatrix} r_{n_L\sigma_L,n'_L\sigma'_L} & t'_{n_L\sigma_L,n'_R\sigma'_R} \\ t_{n_R\sigma_R,n'_L\sigma'_L} & r'_{n_R\sigma_R,n'_R\sigma'_R} \end{pmatrix} \begin{pmatrix} a^L_{n'_L\sigma'_L} \\ b^R_{n'_R\sigma'_R} \end{pmatrix}, \tag{3}$$

where a (b) denotes the right-going (left-going) wave amplitudes, and L (R) denotes the left (right) superconductor. In the labeling of the propagating modes, $n_{L(R)}=e$ for electron-like or h for holelike quasiparticle, $\sigma_{L(R)}$ denotes spin. All the propagating modes are normalized with their probability current. Here $r_{n_L\sigma_L,n'_L\sigma'_L}$ represents the reflection amplitudes for quasiparticles incident from the mode $n'_L\sigma'_L$ into an outgoing mode $n_L\sigma_L$ in the left superconductor. The other three blocks in the scattering matrix have similar meaning. In writing Eq. (3) we adopted the Einstein's sum rule. For an energy $E>\Delta$, the probability current conservation leads to the unitary condition of the S matrix, $S^\dagger S=1$, which gives

$$\begin{pmatrix} a^L_{n_L\sigma_L} \\ b^R_{n_R\sigma_R} \end{pmatrix} = \begin{pmatrix} r^*_{n'_L\sigma'_L,n_L\sigma_L} & t^*_{n'_R\sigma'_R,n_L\sigma_L} \\ t'^*_{n'_L\sigma'_L,n_R\sigma_R} & r'^*_{n'_R\sigma'_R,n_R\sigma_R} \end{pmatrix} \begin{pmatrix} b^L_{n'_L\sigma'_L} \\ a^R_{n'_R\sigma'_R} \end{pmatrix}. \tag{4}$$

For the 2DEG layer, the Hamiltonian for the electron is

$$H_e = \epsilon_k - \alpha(k_x\sigma_y - k_y\sigma_x) - \beta(k_x\sigma_x - k_y\sigma_y) + h(\sigma_x \sin\theta_m \cos\varphi_m + \sigma_y \sin\theta_m \sin\varphi_m + \sigma_z \cos\theta_m).$$

When only the SOI is present, i.e., $h=0$, the TRS is preserved in the system. We can write down the propagating modes in the left and right superconductors as follows:

$$\varphi_{e\uparrow} = (u_\pm \quad 0 \quad 0 \quad v_\mp)^T, \quad \varphi_{e\downarrow} = (0 \quad u_\pm \quad -v_\mp \quad 0)^T,$$

$$\varphi_{h\uparrow} = (0 \quad v_\pm \quad -u_\mp \quad 0)^T, \quad \varphi_{h\downarrow} = (v_\pm \quad 0 \quad 0 \quad u_\mp)^T,$$

where $u_\pm = \sqrt{(1+\Omega/E)/2}e^{\pm i\varphi/4}$, $v_\pm = \sqrt{(1-\Omega/E)/2}e^{\pm i\varphi/4}$ with $\Omega=\sqrt{E^2-\Delta^2}$ and the upper and lower symbols of $\pm$ and $\mp$ denoting, respectively, the left and right superconductors.

The time-reversal operation $T=-i\sigma_y K$ (where K is the complex-conjugation operator) changes $\varphi^{\lambda\pm}_{n\sigma}(\varphi,\theta)$ to $\rho_n\sigma\varphi^{\lambda\mp}_{n\bar{\sigma}}(-\varphi,-\theta)$, where $\lambda=L$ for the left superconductor or $\lambda=R$ for the right one, + (−) denotes right-going (left-going) propagating mode, $\rho_e=1$ and $\rho_h=-1$, and $\sigma=\pm 1$ and $\bar{\sigma}=-\sigma$. In the transformed state $T\psi_\lambda$, the right-going and left-going components for the mode $n_\lambda\sigma_\lambda$ are $\rho_{n_\lambda}\bar{\sigma}_\lambda b^{\lambda*}_{n_\lambda\bar{\sigma}_\lambda}$ and $\rho_{n_\lambda}\bar{\sigma}_\lambda a^{\lambda*}_{n_\lambda\bar{\sigma}_\lambda}$, respectively. Similar to Eq. (3), we can have

$$\begin{pmatrix} \rho_{n_L}\bar{\sigma}_L a^{L*}_{n_L\bar{\sigma}_L} \\ \rho_{n_R}\bar{\sigma}_R b^{R*}_{n_R\bar{\sigma}_R} \end{pmatrix} = \begin{pmatrix} r_{n_L\sigma_L,n'_L\sigma'_L} & t'_{n_L\sigma_L,n'_R\sigma'_R} \\ t_{n_R\sigma_R,n'_L\sigma'_L} & r'_{n_R\sigma_R,n'_R\sigma'_R} \end{pmatrix} \times \begin{pmatrix} \rho_{n'_L}\bar{\sigma}'_L b^{L*}_{n'_L\bar{\sigma}'_L} \\ \rho_{n'_R}\bar{\sigma}'_R a^{R*}_{n'_R\bar{\sigma}'_R} \end{pmatrix}. \tag{5}$$

Comparison of Eqs. (4) and (5) leads to

$$r_{n'_L\sigma'_L,n_L\sigma_L}(\varphi,\theta) = \rho_{n_L}\sigma_L\rho_{n'_L}\sigma'_L r_{n_L\bar{\sigma}_L,n'_L\bar{\sigma}'_L}(-\varphi,-\theta)$$

from which we can obtain the following relations:

$$r_{h\downarrow e\uparrow}(\varphi,\theta) = r_{e\downarrow h\uparrow}(-\varphi,-\theta),$$

$$r_{h\uparrow e\downarrow}(\varphi,\theta) = r_{e\uparrow h\downarrow}(-\varphi,-\theta). \tag{6}$$

From the combination of Eqs. (1), (2), and (6), we can have the conclusion that the total anomalous Josephson current is zero $[J(0)=0]$ when there is only SOI. The conclusion is expected as the breaking of TRS is required for a nonzero anomalous Josephson current.

When there is only Zeeman field, i.e., $\alpha=\beta=0$, $h\neq 0$, the TRS is broken. However, there is another symmetry which prevents the existence of an anomalous Josephson current. This symmetry is related to the combined operation of the time-reversal operator and a spin rotation operator. For the Zeeman field in the direction (θ_m,φ_m), the spin operator included in the combined symmetry operation should be chosen along a direction perpendicular to (θ_m,φ_m), for example, $(\theta_m+\frac{\pi}{2},\varphi_m)$. The combined operator can be written as $(\sigma_x \cos\theta_m \cos\varphi_m+\sigma_y \cos\theta_m \sin\varphi_m-\sigma_z \sin\theta_m)T$ which commutes with the Hamiltonian of electron. The full operator should be constructed in the electron-hole space as follows:

$$\begin{pmatrix} \sigma_n T & 0 \\ 0 & -\sigma_n^* T \end{pmatrix}, \tag{7}$$

where $\sigma_n=\sigma_x \cos\theta_m \cos\varphi_m+\sigma_y \cos\theta_m \sin\varphi_m-\sigma_z \sin\theta_m$. The form in Eq. (7) can ensure that the full operator commutes with the Hamiltonians of the superconductors where the electron-hole coupling Δ is present. For example, if the Zeeman field is along the x direction, i.e., $\theta_m=\frac{\pi}{2}$, $\varphi_m=0$, the combined operator can be chosen to be $\left(\begin{smallmatrix}\sigma_z & 0\\ 0 & -\sigma_z\end{smallmatrix}\right)T$. The operation of the spin operators in the x,y,z direction on the propagating modes are as follows:

$$\begin{pmatrix} \sigma_x & 0 \\ 0 & -\sigma_x \end{pmatrix}\varphi^{\lambda\pm}_{n\sigma} = \varphi^{\lambda\pm}_{n\bar{\sigma}},$$

$$\begin{pmatrix} \sigma_y & 0 \\ 0 & \sigma_y \end{pmatrix} \varphi_{n\sigma}^{\lambda\pm} = \rho_n \sigma \varphi_{n\bar{\sigma}}^{\lambda\pm},$$

$$\begin{pmatrix} \sigma_z & 0 \\ 0 & -\sigma_z \end{pmatrix} \varphi_{n\sigma}^{\lambda\pm} = \rho_n \sigma \varphi_{n\sigma}^{\lambda\pm}. \tag{8}$$

We have $\left(\begin{smallmatrix} \sigma_z & 0 \\ 0 & -\sigma_z \end{smallmatrix}\right) T\varphi_{n\sigma}^{\lambda\pm}(\varphi,\theta) = -\varphi_{n\bar{\sigma}}^{\lambda\mp}(-\varphi,-\theta)$. In the transformed state, the right-going and left-going components for the mode $n_\lambda\sigma_\lambda$ are $-b_{n_\lambda\bar{\sigma}_\lambda}^{\lambda*}$ and $-a_{n_\lambda\bar{\sigma}_\lambda}^{\lambda*}$, respectively. Then we can obtain an expression similar to Eq. (5)

$$\begin{pmatrix} -a_{n_L\bar{\sigma}_L}^{L*} \\ -b_{n_R\bar{\sigma}_R}^{R*} \end{pmatrix} = \begin{pmatrix} r_{n_L\sigma_L,n'_L\sigma'_L} & t'_{n_L\sigma_L,n'_R\sigma'_R} \\ t_{n_R\sigma_R,n'_L\sigma'_L} & r'_{n_R\sigma_R,n'_R\sigma'_R} \end{pmatrix} \times \begin{pmatrix} -b_{n'_L\bar{\sigma}'_L}^{L*} \\ -a_{n'_R\bar{\sigma}'_R}^{R*} \end{pmatrix}, \tag{9}$$

which also leads to Eq. (6) in comparison with Eq. (4). So the total anomalous Josephson current is zero. For other directions of the Zeeman field, we can find the corresponding symmetry operator and prove in a similar way that the anomalous supercurrent is zero.

When both SOI and a Zeeman field are present, the anomalous supercurrent is possible if the Zeeman field is suitably oriented. When only RSOI exists, the numerical results show that the anomalous supercurrent is zero if the Zeeman field points along the x or z axis. We can still understand it in terms of symmetry. If the Zeeman field points along the x or z axis, the system has the symmetry of $\sigma_y R_y T$, where R_y is the reflection transformation about the x axis, i.e., $y \to -y$. $\sigma_y R_y T$ changes $\varphi_{n\sigma}^{\lambda\pm}(\varphi,\theta)$ to $-\varphi_{n\sigma}^{\lambda\mp}(-\varphi,\theta)$, the right-going and left-going components for the mode $n_\lambda\sigma_\lambda$ are $-b_{n_\lambda\sigma_\lambda}^{\lambda*}$ and $-a_{n_\lambda\sigma_\lambda}^{\lambda*}$, respectively. Similar we obtain

$$\begin{pmatrix} -a_{n_L\sigma_L}^{L*} \\ -b_{n_R\sigma_R}^{R*} \end{pmatrix} = \begin{pmatrix} r_{n_L\sigma_L,n'_L\sigma'_L} & t'_{n_L\sigma_L,n'_R\sigma'_R} \\ t_{n_R\sigma_R,n'_L\sigma'_L} & r'_{n_R\sigma_R,n'_R\sigma'_R} \end{pmatrix} \times \begin{pmatrix} -b_{n'_L\sigma'_L}^{L*} \\ -a_{n'_R\sigma'_R}^{R*} \end{pmatrix}, \tag{10}$$

which leads to the following relation in comparison with Eq. (4):

$$r_{n'_L\sigma'_L,n_L\sigma_L}(\varphi,\theta) = r_{n_L\sigma_L,n'_L\sigma'_L}(-\varphi,\theta),$$

i.e.,

$$r_{h\downarrow e\uparrow}(\varphi,\theta) = r_{e\uparrow h\downarrow}(-\varphi,\theta),$$

$$r_{h\uparrow e\downarrow}(\varphi,\theta) = r_{e\downarrow h\uparrow}(-\varphi,\theta). \tag{11}$$

Therefore the anomalous supercurrent is also zero. Similarly when only DSOI exists, the anomalous supercurrent is zero due to the symmetry related to the operator $\sigma_x R_y T$ if the Zeeman field points along the y or z axis.

In general, if both RSOI and DSOI are present, the anomalous Josephson current is nonzero except for the Zeeman field along the z axis. In the special case of $\alpha=\beta$, the situation is subtle. In this case, the Hamiltonian for the SOI is $H_{\mathrm{SOI}}=-\alpha(k_x-k_y)(\sigma_x+\sigma_y)$. We can see that the eigen spin axis of SOI is always pointing to $(\frac{\pi}{2},\frac{\pi}{4})$. According to the above discussion, the Zeeman field cannot point to a direction which is perpendicular to $(\frac{\pi}{2},\frac{\pi}{4})$; otherwise the symmetry $(\sigma_x+\sigma_y)T$ arises and leads to a zero anomalous supercurrent. Moreover, the anomalous supercurrent is also zero if the Zeeman field is parallel to $(\frac{\pi}{2},\frac{\pi}{4})$. It is probably due to some higher symmetry in the expression of Josephson current which cannot be seen in the Hamiltonian. Thus the anomalous supercurrent will be zero if the angle between the Zeeman field and the eigenspin axis of SOI $(\frac{\pi}{2},\frac{\pi}{4})$ is 0 or $\frac{\pi}{2}$, as shown in Fig. 2.

Our symmetry analysis shows that the appearance of an anomalous Josephson current is possible only when the symmetries related to T, $\sigma_n T$, and $\sigma_n R_y T$ are all broken. This symmetry criterion is general and not limited to the particular model used here because only the two s-wave superconductor leads and the symmetries of the interlayer have effects in the symmetry analysis of the s-matrix elements. So the symmetry criterion for the anomalous supercurrent are useful guidelines for orienting the Zeeman field as well as designing new junctions with anomalous supercurrent and superconducting rectifying behavior. For example, in another interesting junction where two conventional s-wave superconductor leads are connected with a ferromagnetic trilayer,[11,12] our symmetry criterion indicates that the ferromagnetic trilayer should have noncoplanar magnetizations. If the magnetizations are coplanar, the symmetry $\sigma_{\mathbf{n}} T$ remains and prevents the anomalous supercurrent from appearing where $\mathbf{n}$ is the direction vector perpendicular to the magnetization plane. Besides, the situation of the S/F/S junction on the surface of topological insulator is similar to the case of SOI. The formalism and the results of the symmetry analysis on the anomalous supercurrent keep unchanged.

IV. CONCLUSION

In summary, we studied the existence conditions for the anomalous Josephson effect using a formalism in which the Josephson current is related to the Andreev reflection coefficients. We found that an anomalous Josephson current can be produced by breaking some symmetries of the Hamiltonian. These symmetries impose symmetry conditions on the Andreev reflection amplitudes and make the anomalous Josephson current zero even when time-reversal symmetry is broken by a Zeeman field. The present formalism and the principles are not just limited to 2DEG junctions but also applicable to conventional superconductor junctions coupled with other structures.

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